Convex Geometry

1st midterm - SAMPLE

1) Consider the polynomial $p(t) = \sum_{i=0}^{n} a_i t^i$ as a point $(a_0, a_1, \ldots, a_n) \in \mathbb{R}^{n+1}$. Let

$$V = \{p \in \mathbb{R}^{n+1} : p(t) \geq 0 \text{ for every } t \in \mathbb{R}\}.$$

Prove that V is a closed, convex set.
(5 points)

2) Let K be the triangle with vertices $(0,0), (0,1)$ and $(1,0)$, and let L be the reflection of K to the origin. Determine the sets $K + L$ and $K - L$.
(5 points)

3) Show that if $S \subseteq \mathbb{R}^2$ is an arbitrary nonempty set and $p \in \operatorname{int}\operatorname{conv} S$, then there are points $p_1, p_2, p_3, p_4 \in S$ such that $p \in \operatorname{int}\operatorname{conv}\{p_1, p_2, p_3, p_4\}$.
(5 points)

4) Prove that if $K \subset \mathbb{R}^n$ is convex, then $K + K = 2K$. Give an example showing that the same property does not hold if we drop the condition that K is convex.
(5 points)