Convex geometry
Assignment

Problem 1. *Prove that if $K \subseteq \mathbb{R}^n$ is closed, convex and not bounded, then for every $p \in K$, K contains a half line starting at p.*

Problem 2. *Let $K \subseteq \mathbb{R}^n$ be a closed, convex set. Prove that if K contains a half line L starting at p, then for any $q \in K$, K contains the half line starting at q and having the same direction as L.*

Problem 3. *Let $K \subseteq \mathbb{R}^n$ be the convex hull of the points whose every coordinate is 1 or -1, i.e. K is an origin-symmetric cube of edge length 2. Define K as the intersection of finitely many closed half spaces.*

Problem 4. *Let $e_1, \ldots, e_n$ be the standard basis vectors of $\mathbb{R}^n$. Let $K = \operatorname{conv}\{\pm e_1, \pm e_2, \ldots, \pm e_n\}$. Define K as the intersection of finitely many closed half spaces.*

Problem 5. *Let $X \subset \mathbb{R}^n$ be a compact set. Let H be an open half space containing X. Prove that then there is a closed half space $H_0 \subset H$ such that $X \subset H_0$. Is this statement true if X is not necessarily closed?*

Problem 6. *A compact, convex set $K \subset \mathbb{R}^n$ is called a set of* constant width *d, if for any unit vector u, the distance of the pair of supporting hyperplanes perpendicular to u is d. Prove that a compact, convex set K is of constant width d if and only if $h_K(u) + h_K(-u) = d$ for every unit vector u.*

Problem 7. *Let T be a regular triangle of edge length $a > 0$. The intersection of the three closed disks of radius a centered at the vertices of T is called a* Reuleaux triangle*. Prove that this set is a set of constant width a.*

Problem 8. *Let R be the Reuleaux triangle defined in the previous problem. Prove or disprove that the set $R + \lambda B^2$ is a set of constant width for every $\lambda \in \mathbb{R}$. (B^2 is the closed unit disk centered at the origin.)*

Problem 9. *Let T be a regular tetrahedron in $\mathbb{R}^3$ of edge length a. Let R denote the intersection of the four closed balls of radius a centered at the vertices of T. Prove that R is not a set of constant width a.*