19.05.2023

Convex Geometry

Midterm 2

1) Recall that a set $K \subseteq \mathbb{R}^n$ is called a convex cone if it is convex, and for any $x \in K$ and $\lambda \geq 0$, $\lambda x \in K$. Prove that a convex cone has at most one exposed point. (5 points)

2) Let $K \subset \mathbb{R}^n$ be a closed, convex set. Prove that for any $q \in \mathrm{bd}(K)$, there is a hyperplane H in $\mathbb{R}^n$ separating q from K. (5 points)

3) Let $S \subset \mathbb{R}^3$ be the boundary of a regular tetrahedron. What is the Euler characteristic of S? (5 points)

4) Consider the closed segment $A = \{(0, 0, t) \in \mathbb{R}^3 : -1 \leq t \leq 1\}$, and the circular disk $B = \{(x, y, 0) \in \mathbb{R}^3 : (x - 1)^2 + y^2 \leq 1\}$. Let $K = \mathrm{conv}(A \cup B)$. Prove that K is a compact, convex set and compute $\mathrm{ext}(K)$. Is $\mathrm{ext}(K)$ closed? (5 points)