

BSC IN MATHEMATICS FINAL EXAM
Algebra, Analysis, Discrete Mathematics, Geometry, Probability Theory
Valid from Summer 2026

ALGEBRA

1. Number theory

Divisibility, unit, definition of greatest common divisor. Quotient-remainder theorem, Euclidean algorithm. Irreducible number, prime number, fundamental theorem of number theory. Definition of congruence, elementary properties, residue class, complete and reduced residue system. Fermat–Euler theorem.

2. System of linear equations and their matrices

Homogeneous and inhomogeneous system of linear equations, matrix rank, condition of solvability, number of solutions. \mathbb{R}^n , vector space, subspace, linear independence, linear dependence, basis, dimension. Matrix row, column and null space. Gaussian elimination, elementary row operations, row echelon form, reduced row echelon form. Matrix operations and their properties, matrix inverse, transpose, pseudoinverse. The existence and application of LU and PLU decompositions.

3. Linear transformations, matrices

Concept of linear map, matrix form, change of basis. Eigenvalue, eigenvector, diagonalization. Cayley–Hamilton theorem and Jordan’s normal form. The Euclidean scalar product and norm. Orthogonal and symmetric matrices. Definiteness of quadratic forms. Scalar product over complex numbers and finite fields. Bilinear forms, standard form, principal axis theorem. Matrix decompositions: SVD, QR, spectral decomposition.

4. Group theory

Group, subgroup, normal subgroup, factor group, homomorphism, homomorphism theorem. Order of a group, of a group element, Lagrange’s theorem, Cauchy’s theorem. Subgroups of a cyclic group. Fundamental theorem of finite Abelian groups, p -groups, Sylow subgroup, Sylow theorems.

5. Polynomial rings and fields

Ring, subring, ideal, factor ring. Field, characteristic, finite fields. In $F[x]$ and \mathbb{Z} : division with remainder, principal ideal ring, maximum ideals and factorstaken with them. Complex numbers, polynomials, the fundamental theorem of algebra. Degree of field extension, algebraic element, simple extension.

ANALYSIS

1. Differentiation

For single-variable scalar functions: derivative of a function, differentiation rules, mean value theorems (Rolle and Lagrange), relationship between monotonicity and the derivative, and between convexity and the second derivative. Hölder and Minkowski inequalities. For multivariable vector-valued functions: partial derivative, derivative of a function, differentiation rules. Jacobian matrix and Jacobian determinant of a function. Directional derivative. Gradient, divergence, curl, and Laplace operator of a function. Tangent plane. Local (strict) maximum/minimum of a function. Characterization of local extrema using derivatives. Local maximum and minimum under constraints. Necessary condition for the existence of conditional extrema. Lagrange multipliers.

2. Function Series and Integration

Convergence of real series. Convergence criteria (majorant/minorant, root and ratio tests). Leibniz series. Pointwise, uniform, and locally uniform convergence of function series. Power series and the Cauchy–Hadamard theorem. Term-by-term integrability and differentiability of scalar-scalar function series. Taylor series expansion. Upper and lower sums and oscillation sum with respect to a given partition. Riemann integrability. Newton–Leibniz theorem. Applications of integral calculus (arc length of a curve; area of a plane figure; volume and surface area of a solid of revolution). Polar, cylindrical, and spherical coordinates. Intuitive introduction to integration over planar and spatial domains. Integration over normal domains, writing two- and three-dimensional integrals as successive one-dimensional integrals. Changing the order of integration. Normal vector and surface area of a surface. Volume of a spatial domain. Gauss–Ostrogradsky theorem and Stokes’ theorem.

3. Complex Analysis and Fourier Series

Differentiability and derivative of complex functions. Cauchy–Riemann equations. Contour integral. Newton–Leibniz theorem. Goursat’s lemma. Index function of a curve and its properties. Cauchy’s theorem on integrals over homotopic contours. Cauchy integral formulas. Analyticity of holomorphic functions. Liouville’s theorem and the fundamental theorem of algebra. Properties of roots of holomorphic functions and multiplicity of roots. Local maximum principle. Laurent series expansion. Singularities of a function, poles of order n , and essential singularities. Residue theorem. Fourier coefficients and Fourier series expansion. Convergence of Fourier coefficients for integrable functions. Convergence of the Fourier series for twice continuously differentiable functions.

4. Metric Spaces and Normed Spaces

Open, closed, bounded, and compact sets and their basic properties. For real numbers: Borel–Lebesgue theorem and Cauchy criterion. In metric spaces: continuity and limit of a function, Cantor’s intersection theorem for compact sets, Borel–Lebesgue theorem, Bolzano–Weierstrass theorem; Weierstrass’s maximum–minimum principle for functions, Heine’s theorem, and Banach fixed-point theorem. Complete metric space, Banach space, and Hilbert space. Cauchy–Schwarz inequality, expansion of a vector in a complete orthonormal system, Parseval’s identity. Operator norm and its properties. Carl Neumann series. Equivalence of norms. Example of a discontinuous linear mapping and non-equivalent norms.

5. Differential Equations

Initial value problem, Lipschitz property, existence and uniqueness theorem. Existence of the solution and its continuous dependence on initial values. Types of explicitly solvable equations (separable, linear and exact differential equations). Solution of systems of linear differential equations. Solution of higher order linear differential equations. Concept of stability and asymptotic stability. Stability analysis by linearization. Lyapunov’s theorems.

GEOMETRY

1. Convex polyhedra

- definition of an n -dimensional convex polyhedron (intersection of half spaces vs. convex hull of the vertices);
- classification of n -dimensional regular polyhedra;
- Euler’s theorem in 3 dimensions, the existence of Platonic solids.

2. Non-Euclidean geometries

- internal angle sum of triangles and area in hyperbolic and spherical geometry (Legendre's theorems);
- distance in non-Euclidean geometries (cross ratio);
- homogeneous coordinates and the real projective plane (Desargues's theorem, Pappos's theorem).

3. Differential geometry of curves and surfaces in 3-dimensional Euclidean space

- The fundamental theorem of curve theory, the fundamental theorem of surface theory, Theorema Egregium and the description of notions included.

DISCRETE MATHEMATICS AND ALGORITHMS

1. Data sorting methods. Basic search methods, their data structures

Selection, insertion, quick, merge sort, lower bound for the number of comparisons in sorting algorithms, bin sort, radix sort. Binary tree traversals, binary search trees, {red-black-trees, 2-3-trees, bucket hashing, open addressing hashing}.

2. Algorithms of graphs

Breadth-first and Depth-first search (BFS and DFS), DAG. Dijkstra, Ford and Floyd Algorithms. Concept of minimum weight spanning trees, greedy algorithm, Kruskal theorem. Concept of bipartite graphs, matching in bipartite graphs, Hall and Frobenius theorem, alternating path algorithm. Network flows, Ford–Fulkerson theorem, augmenting path algorithm.

3. Colorings of graphs, planar graphs

Colorings of graphs, chromatic number, clique number, lower and upper bounds on chromatic number. Line graphs, chromatic index, Vizing theorem, König theorem. Planar graphs, Euler formula, four-color theorem, homeomorphism (topological isomorphism), Kuratowski theorem, Fáry–Wagner theorem. Conditions on the existence of Hamilton cycle.

4. The notion of NP, famous problems in NP. NP completeness

The NP language class, Karp reduction, NP completeness, Cook–Levin theorem, famous NP-complete languages: 3SAT, HAM-CYCLE, HAM-PATH, 3COLOR, MAXCLIQUE, Max Independent Set, SUBSETSUM, PARTITION.

PROBABILITY THEORY

1. Fundamentals of probability theory

Probability space, axioms of probability, Sieve formula (inclusion/exclusion formula). Conditional probability, Bayes's theorem, independency of events. Discrete and absolutely continuous random variables, cumulative distribution function (CDF), function, probability weight function, probability density function (PDF), expected value, variance. Notable distributions: binomial, (optimistic) geometric, Poisson, uniform, exponential, normal distributions. De Moivre–Laplace theorem.

2. Joint distributions, law of large numbers and central limit theorem

Joint distributions, marginal distributions, and conditional distributions. Conditional expected value. Independence, covariance, and correlation. Multidimensional normal distribution. Sums of independent random variables, convolution. Markov's and Chebyshev's inequality, weak law of large numbers. Central limit theorem and its applications.

The parts of the material in curly brackets were not included in the curriculum every year.