Combinatorics and Graph Theory 2 topics

1. **Perfect graphs**, examples for perfect and not perfect graph families Lovász' (weak) Perfect Graph Theorem, the strong perfect graph theorem.

2. **Partially ordered sets (posets)**, chain, antichain, partition into chains and antichains, comparability graphs. Theorems of Mirsky and Dilworth, connection with perfect graphs.

3. **Planar graphs**, plane graphs, graph drawings on the sphere, connection with planarity. Euler's formula, maximum number of edges in a simple planar graph, examples for non-planar graphs, Kuratowski's theorem.

4. **Dual of planar graphs** (geometric dual), properties of the geometric duals, bonds, connection of bonds and cycles in G and the dual of G. Abstract duality, weak isomorphism, Whitney's theorems.

5. **List coloring**, choosability of a graph ($\mathrm{ch}(G)$), connection with the chromatic number and the maximum degree. Choosability of planar graphs (Thomassen's theorem and its sharpness).

6. **Ramsey theory:** Ramsey-numbers, Erdős–Szekeres upper bound (recursive and explicit), Erdős' unconstructive lower bound for $R(k,k)$. Generalized Ramsey numbers: multicolor Ramsey numbers, Ramsey numbers of general graphs ($R(G,H)$). Ramsey type results: Schur's Theorem, Van der Waerden's Theorem.

7. **Extremal graph theory:** $\mathrm{ex}(n;H)$, Turán graphs, Turán's theorem about $\mathrm{ex}(n,K_r)$. Erdős–Stone(–Simonovits)-theorem (the Fundamental Theorem of Extremal Graph Theory). Upper bound for $\mathrm{ex}(n;C_4)$, construction of C_4-free graphs with many edges from finite projective planes.

8. **Extremal set systems (hypergraphs):** Sperner's Theorem, Erdős–Ko–Rado-Theorem, Fisher's Inequality. The dual of a hypergraph, Erdős–De Bruijn Theorem.

9. **Catalan numbers, generating functions:** Combinatorial definition, recurrence relation, explicit formula. Generating function of Catalan numbers.