Convex geometry
Topics for the final exam

1. **Affine subspace, affine and convex combination, convex hull.** affine subspaces and their intersections, hyperplanes and half spaces, affine hull, relative interior, relative boundary, affine combination, affine dependence and independence, convex combination, segment, convex set, closed convex sets and closed half spaces, convex hull, characterization of convex hull via convex combinations
2. **Fundamental theorems and applications.** Radon's theorem, Carathéodory's theorem, convex hull of compact set, colorful Carathéodory theorem, finite and infinite Helly theorems, Jung's theorem
3. **Minkowski sum and support function.** Minkowski sum, Minkowski sum of convex sets, support function and its properties, reconstructing a convex set from its support function, support function of a Minkowski sum
4. **Separation and isolation theorems.** separation, strict separation, isolation, strict isolation, projections and their properties, isolation theorem for open convex sets, interiors of convex sets, general isolation theorem, separation of disjoint convex sets, strict separation of disjoint compact and closed convex sets
5. **Faces, extremal and exposed points.** supporting half space and supporting hyperplane, face, exposed and extremal points, linear functionals and extremal points of faces, Krein–Milman theorem, Straszevicz theorem
6. **Valuations and the Euler characteristic.** indicator function, inclusion-exclusion formula, algebras of compact/closed convex sets, valuations, Euler characteristic and its properties
7. **Convex polytopes and polyhedral sets.** convex polytope, minimal representation, exposed and extremal points of convex polytopes, faces of convex polytopes, intersections of faces of compact convex sets, faces of faces of compact convex sets and polytopes, polyhedral set, characterization of bounded polyhedral sets
8. **Face structures of polytopes.** face lattice and its properties, atoms of the face lattice, Euler characteristic of the boundary of a convex polytope, f-vector, boundary points of polytopes, Euler's theorem
9. **Polarity.** polar, basic properties of polar sets, compactness of the polar, polar of the polar, faces of compact convex sets and those of their polars, dual polytopes, volume product, Blaschke–Santaló theorem, Mahler's conjecture
10. **Hausdorff distance and Banach–Mazur distance.** Hausdorff distance, support functions and the Hausdorff distance of compact convex sets, metric space of compact convex sets, completeness, Blaschke's selection theorem, approximation by polytopes, Banach–Mazur distance
11. **Ellipsoids.** ellipsoid, parametrizations, volume, maximum-volume ellipsoid, relation to containing ellipsoid, implications for Banach–Mazur distance, Löwner–John ellipsoids
12. **Lattices and convex bodies.** lattice, fundamental parallelotope, basis, determinant of basis change map, determinant of a lattice, lattice-translates of fundamental parallelotope, asymptotic growth of number of lattice points in scaled convex bodies, Blichfeldt theorem, Minkowski's convex body theorem, application to rational approximation, four-square theorem