

Partial Differential Equations 1

Final exam questions for BSc students

2026

1. **Basics.** Definition of a PDE, well-posedness. Second-order linear PDEs, their classification, examples: Poisson equation, heat equation, wave equation.
2. **Definition of distributions.** The set $\mathcal{D}(\Omega)$, definition of a distribution (using the sequential continuity and the boundedness properties, their connection). Regular distribution, Dirac-delta.
3. **Derivative of distributions.** The value of $T_{\partial^\alpha f}$, the derivative of a general distribution. The derivative of a regular distribution corresponding to a one-variable piecewise smooth function.
4. **Convolution of distributions.** The value of $T_{f \times g}$, the direct product of distributions. The value of $T_{f * g}$, the convolution of distributions. Commutativity, associativity.
5. **Fundamental solutions.** Definition of a fundamental solution, its connection to the solution of a linear PDE. The application of this statement in the case of the wave equation.
6. **Wave equation.** Classical solution of the wave equation (defined on $(t, x) \in \mathbb{R}_0^+ \times \mathbb{R}^n$). General (weak) solutions of this equation (in the distributional sense). The connection between the classical and the weak solutions. Properties: number of classical solutions, speed of the wave propagation.
7. **Heat equation.** Classical solution of the heat equation (defined on $(t, x) \in \mathbb{R}_0^+ \times \mathbb{R}^n$). General (weak) solutions of this equation (in the distributional sense). The connection between the classical and the weak solutions. Properties: number of classical solutions, speed of the heat propagation.
8. **Boundary-value problems (BVPs).** Classical first, second, and third-type BVPs. Theorem about the number of solutions.
9. **Eigenvalue problems.** Eigenvalue problems (first, second, third type). Theorem about operator L and its eigenvalues and eigenfunctions. The method of Fourier.

10. **Sobolev spaces.** Definitions of $H^k(\Omega)$ and $H_0^k(\Omega)$ (completion, general derivatives, their connections). Equivalent norms of $H_0^1(\Omega)$.
11. **Weak solutions of BVPs.** Weak form of BVPs. Connection between the classical and weak forms of BVPs. Theorem about the solution of a weak BVP, connection to the classical BVP.