

Probability Theory 2. - Topics for exam

1. **Convolution I.** Definition of discrete convolution, properties, examples: $\text{BIN} * \text{BIN}$, $\text{POI} * \text{POI}$, $\text{GEO} * \text{GEO}$, definition of Stieltjes integral, definition of convolution of distributions, basic properties.
2. **Convolution II.** Smoothing theorem of convolution, Gamma-distribution, connection with the Poisson- and Exponential distributions, CLT for exponential distribution, Euler's Gamma-functions, basic properties, Chi^2 distribution.
3. **Prob. generating function I.** Definition of probability generating function, basic properties, examples (BIN , POI , GEO). Moments, reconstruction of the distribution from the generator function. Convolution, mixed distribution, prob. generating function of randomly many terms,
4. **Prob. generating function II.** Branching (Galton-Watson) processes, probability of extinction, simple random walk on \mathbf{Z} , hitting times, recurrence, transience.
5. **Concentration inequalities I.** Markov's, Chebisev's, Paley-Zigmund's, Cantelli's inequalities, weak law of large numbers (with second moment), Coupon-collector problem.
6. **Concentration inequalities II.** Chernoff's, Hoeffding's, Bernstein's inequalities, Cramér's bound, logarithmic moment generating function, Legendre-transform, and basic properties.
7. **Types of convergences.** Almost sure (strong) convergence, Convergence in probability (Stochastic conv.), convergence in L^1 , L^2 , L^p . Relations between them. (strong \Rightarrow in prob., in prob. \Rightarrow strong on subsequence, $L^1 \Leftrightarrow$ in prob. + uniformly integrability, counterexamples), bounded convergence theorem, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.
8. **Strong Law of Large Numbers I.** Borel-Cantelli Lemmas. Strong law of large numbers with fourth moment, Kolmogorov's inequality, and Kolmogorov's two series.
9. **Strong Law of Large Numbers II.** Kronecker's lemma, Strong law of large numbers with first moment, tail σ -algebra, Kolmogorov 0-1 law, Hausdorff's Theorem, Hardy & Littlewood's Theorem.
10. **Characteristic function I.** Definition, basic properties (bounds, uniform continuity, positive definite), Bochner's Theorem (without proof), connection with lattice distributed random variables, characteristic function of notable distributions (EXP , UNI , normal). Cauchy distribution and its characteristic function.
11. **Characteristic function II.** Moments and the derivatives of the characteristic function, smoothness of the distribution function and the decay of the characteristic function at $\pm\infty$, Dirichlet's and Riemann-Lebesgue's Lemma.
12. **Weak convergence of distributions I.** Reconstruction of the distribution from the characteristic function, definition of weak convergence of probability distributions (with probability measures), Weak convergence of distribution and pointwise convergence of distribution functions.
13. **Weak convergence of distributions II.** Tightness and Prohorov's theorem, Lévy's lemma and continuity theorem, central limit theorem.