Topics for the final exam in Tools of Modern Probability Theory

1. Gaussian integrals; spherically symmetric integrals. Application: expressing the surface of hyperspheres using the Euler gamma function.
2. Measure theoretic language in probability: measure and probability; distribution of random variables; integral and expectation; expectation of a random variable vs. expectation of a probability distribution, calculation of the expectation as a sum or a Riemannian integral in special cases.
3. Exchanging the integral and the limit: monotone convergence theorem, dominated convergence theorem, Fatou's lemma. Examples and applications.
4. Product space, product measure. Exchanging integrals: Fubini's theorem. Examples and applications.
5. Hilbert spaces – completeness of L^2.
6. Riesz representation theorem (about linear forms on Hilbert spaces).
7. Absolute continuity of measures. Radon-Nikodym theorem.
8. Conditional expectation of random variables. Motivation, definition, existence, uniqueness, basic properties. Calculation in simple examples.